Influence of Information Flow Topology on Closed-loop Stability of Vehicle Platoon with Rigid Formation*

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Abstract—Besides automated controllers, the information flow among vehicles can significantly affect the dynamics of a platoon. This paper studies the influence of information flow topology on the closed-loop stability of homogeneous vehicular platoon moving in a rigid formation. A linearized vehicle longitudinal dynamic model is derived using the exact feedback linearization technique, which accommodates the inertial delay of powertrain dynamics. Directed graphs are adopted to describe different types of allowable information flow interconnecting vehicles, including both radar-based sensors and V2V communications. Under linear feedback controllers, a unified closed-loop stability theorem is proved by using the algebraic graph theory and Routh–Hurwitz stability criterion. The theorem explicitly establishes the stabilization threshold of linear controller gains for platoons with a large class of different information flow topologies. Numerical simulations are used to illustrate the results.

I. INTRODUCTION

Platooning of road vehicles provides a promising solution to several critical issues of today's road transportation systems due to its potential to significantly increase highway capacity, enhance safety, and reduce fuel consumption, as well as CO2 emission [1]. The objective of platoon control is to ensure that all vehicles in a platoon move at the same speed while maintaining a desired formation geometry, which is specified by a desired inter-vehicle spacing policy. Control design of a platoon has a long history that dates back to the mid-sixties of the last century [2]. It has recently attracted extensive research interests, see [3-8] and the references therein.

From the viewpoint of control, a platoon system can be considered as a combination of vehicle longitudinal dynamics, information exchange flow, decentralized controllers and inter-vehicle spacing policies [4][5][7]. The vehicle longitudinal dynamics depicts the behavior of each vehicle in longitudinal direction. The platoon is said to be "homogeneous" if all vehicles have identical dynamics; otherwise it is called "heterogeneous" [8]. The information exchange flow defines how the vehicles in a platoon exchange information with each other, including the exchanged information and information flow topologies among vehicles. Decentralized controllers implement specific feedback control laws for each vehicle. Most common control laws in the literature are linear, for comprehensive results on theoretical analysis and design methods, and convenience in hardware implementation [5][8]. The available information to each controller is often limited to a neighboring region because of the range limitation of sensing and communication systems. As a result, controllers use only local information to achieve a global performance for the platoon. The spacing policy sets rules of the desired distance between two adjacent vehicles and further dictates the desired formation geometry for the platoon. Here, we focus on a homogenous vehicle platoon with rigid formation and linear feedback controllers, as used by Seiler [7], Barooah [9], and Yadlapalli [10].

The information flow topology applied in a platoon is closely related to the way a vehicle acquires the information of its surrounding vehicles. Early-stage platoons are mainly radar-based without widely using the inter-vehicle communications. This means that a vehicle can only obtain the information of its nearest neighbors, i.e. the front and back vehicles [5][11][12]. Under the radar-based sensing framework, the commonly used information flow topologies include the predecessor following type, the predecessor-leader following type, and the bidirectional type [3][7][11-13]. Note that the predecessor-leader following type needs a leader with information broadcasting functions. Their relationship with string stability was studied by Darbha and Hedrick [4][15], Seiler [7], and Khatir [14] etc. Darbha et al. pointed out that under the constant-distance policy, a predecessor following-type platoon with identical linear controllers cannot guarantee string stability because its associated denominator polynomial has at least an instability root [4]. Seiler et al. showed that there was an essential limitation with localized linear controllers using the constant distance policy and predecessor following type since small spacing errors acting on one vehicle can be amplified along the vehicle string due to a complementary sensitivity integral constraint [7]. Four major approaches have been proposed to improve string stability of a platoon. One approach is to use non-identical controllers to achieve bounded stability, but at the expense that the controller gains must increase linearly with respect to the platoon scale [14]. The second approach is to broadcast the leader information to every following vehicle, resulting in the aforementioned predecessor-leader following topology [4]. This topology inevitably introduces certain time

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delays because it needs to transmit information from the leader to all the following vehicles. The third approach is to relax the formation rigidity of a platoon by using the constant time headway policy instead of the constant distance policy [15][16]. The last approach is to extend the information flow topology to the bidirectional type [5][7][12][17-20]. In this approach, two radars are installed on each vehicle, front and back, to detect its adjacent two vehicles. The controller then can use the information of both its preceding vehicle and following vehicle in its control strategy.

Although extensive research has been conducted on radar-based topologies, more information flow topologies have emerged with the rapid deployment of vehicle-to-vehicle (V2V) communications such as DSRC, VANET, and MANET [21]. V2V communications generate various information flow topologies, including the two-predecessors following type, two-predecessor-leader following type and k-predecessors following type, etc. [22]. A few studies have been conducted to examine their influence on platoon performance, including stability and scalability. For example, Yadlapalli et al. pointed out that at least one vehicle should communicate to a large number of other vehicles if the spacing errors in the platoon need to be guaranteed insensitive to the platoon size [10]. Darbha and Pagilla investigated the limitations of employing undirected information flow to maintain a rigid formation and indicated that there was a critical size of platoon scale beyond which the motion would lose stability [23]. Fax et al. used the eigenvalues of the Laplacian matrix to determine the formation stability and proved that formation stability could be decomposed into two components: i.e. stability of information flow for the given graph and stability of individual vehicles for the given controller [24].

This paper further studies the influence of different information flow topologies on the closed-loop stability of a platoon of homogenous vehicles moving in a rigid formation. The main contribution of this paper is to derive explicitly a unified closed-loop stability theorem by using the algebraic graph theory and Routh–Hurwitz stability criterion. The stability theorem is suitable for a large class of information flow topologies, either radar-based or communication-based. The theorem is actually an extension of the main result in Ghasemi et al.[17][18]. The main result in [17] and [18] was derived from another approach, called partial differential equation approximation, but its application is limited to bidirectional topologies and bidirectional–leader topologies. The remainder of this paper is organized as follows: Section II introduces the problem of platoon control, including graph-based modeling of different types of information flow topologies. Section III presents the closed-loop stability theorem for homogeneous platoons under different information flow topologies. Numerical simulations are shown in Section IV. Section V is for concluding remarks.

II. PROBLEM STATEMENT

The platoon has $N + 1$ vehicles, shown in Fig. 1, including a leading vehicle (noted as the leader) and N following vehicles (noted as followers). The platoon runs on a flat road, and can have different information flow topologies, either radar-based or communication-based. Fig. 1 shows six kinds of commonly used topologies, including:

(1) Predecessor following topology (PF);
(2) Predecessor-leader following topology (PLF);
(3) Bidirectional topology (BD);
(4) Bidirectional-leader topology (BDL);
(5) Two predecessors following topology (TPF);
(6) Two predecessor-leader following topology (TPLF).

For simplicity, many other topologies are not considered here, but they all can be analyzed using similar approaches. Note that the exchanged information can contain all the subjected vehicle's position, velocity, and acceleration or some of them.
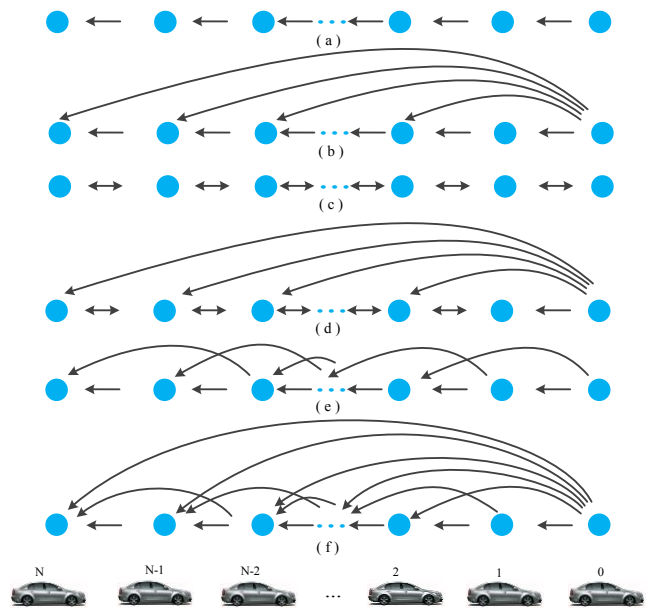

Figure 1.　Typical Information Flow Topologies for Platoon. (a) PF; (b) PLF; (c) BD; (d) BDL; (e) TPF; (f) TPLF

A. Model for Vehicle Longitudinal Dynamics

A platoon can be viewed as a collection of nodes, i.e. vehicles. For each vehicle, its longitudinal dynamics include the engine, drive line, brake system, aerodynamics drag, tire friction, rolling resistance, and gravitational force, etc. Some reasonable assumptions should be used to obtain a concise model for control [16][19][25-27]:

(1) The tire longitudinal slip is negligible, and the powertrain dynamics are lumped into a first-order inertial transfer function;
(2) The vehicle body is considered to be rigid and symmetric;
(3) The influence of pitch and yaw motions is neglected;
(4) The driving and braking torques are controllable inputs.

The vehicle longitudinal dynamics are simplified, but still nonlinear, as follows:

$$\begin{cases} \dot{s}_i(t) = v_i(t) \\ \dot{v}_i(t) = \frac{1}{m_{i,veh}}\left(\eta_{T,i}\frac{T_i(t)}{R_i} - C_{A,i}v_i^2 - m_{i,veh}gf\right), \\ \tau_i\dot{T}_i(t) + T_i(t) = T_{i,des}(t) \end{cases} \quad (1)$$

$$i = 1,2\cdots,N$$

where $s_i(t), v_i(t)$ denote the position and velocity of vehicle i, $m_{i,veh}$ is the vehicle mass, $C_{A,i}$ is the lumped aerodynamic drag coefficient, g is the acceleration due to gravity, f is the coefficient of rolling resistance, $T_i(t)$ denotes the actual driving/braking torque, $T_{i,des}(t)$ is the desired driving/braking torque, τ_i is the inertial delay of vehicle longitudinal dynamics, R_i denotes the tire radius and $\eta_{T,i}$ is the mechanical efficiency of driveline. The position and velocity of the leading vehicle are denoted by $s_0(t)$ and $v_0(t)$, respectively.

The exact feedback linearization technique is used to convert the nonlinear model into a linear one for controller design. The same technique has been widely used before [4][5][12][17] and [18]. The feedback linearization law is

$$T_{i,des}(t) = \frac{1}{\eta_{T,i}}\left(C_{A,i}v_i(2\tau_i\dot{v}_i + v_i) + m_{i,veh}gf + m_{i,veh}u_i\right)R_i, \quad (2)$$

where u_i is the new input signal after linearization. Then, we obtain a linear model for vehicle longitudinal dynamics

$$\tau_i\dot{a}_i(t) + a_i(t) = u_i(t), \quad (3)$$

where $a_i(t) = \dot{v}_i(t)$ denotes the acceleration of vehicle i,.

For platoon control, a 3rd-order state space model is derived for each vehicle:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t), \quad (4)$$

where

$$x_i(t) = \begin{bmatrix} s_i \\ v_i \\ a_i \end{bmatrix}, A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix}.$$

B. Model for Information Flow

The information flow topology describes the information used by each local controller and has significant influence on the collective behavior of the platoon. Moreover, some properties (e.g. stability and scalability) are not only related to decentralized controllers, but also depend on the information flow topology [7][10][23] . Here, directed graphs are adopted to develop a unified model for allowable information flow that interconnect vehicles in a platoon, including all aforementioned topologies.

The platoon includes N followers and 1 leader. The information flow among followers is modeled by a directed graph topology $G = \{V, E\}$ with N nodes $V = \{\alpha_1, \alpha_2, \dots, \alpha_N\}$, and a set of edges $E = V \times V$. The node α_i represents the i-th vehicle in a platoon whose dynamics is described by (4), and each edge represents a directional information exchange between two vehicles. To model the information flow from the leader to followers, we define an augmented graph as $\widetilde{G} = \{\widetilde{V}, \widetilde{E}\}$, where $\widetilde{V} = \{\alpha_0, \alpha_2, \dots, \alpha_N\}$ is the node set including both the leader and followers. The properties of information flow modeled by the directed graphs G and $\widetilde{G}$ can be represented by three matrixes:

(1) Adjacent matrix M;
(2) Laplacian matrix L;
(3) Pining matrix P.

The method that uses matrices to study graphs is known as algebraic graph theory [24][28]. The adjacent matrix associated with graph G is defined as $M = [m_{ij}] \in \mathbb{R}^{N \times N}$ with each entry defined as

$$\begin{cases} m_{ij} = 1, & if \ \{\alpha_j, \alpha_i\} \in E \\ m_{ij} = 0, & if \ \{\alpha_j, \alpha_i\} \notin E \end{cases}, \quad (5)$$

where $\{\alpha_j, \alpha_i\} \in E$ means there is a directional edge from vehicle j to vehicle i, i.e. vehicle i can obtain the information on vehicle j. It is assumed that there is no self-loop, i.e. $m_{ii} = 0$. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N \times N}$ associated with graph G is defined as:

$$l_{ij} = \begin{cases} -m_{ij} & , \ i \neq j \\ \sum_{k=1}^{N} m_{ik}, & i = j \end{cases}. \quad (6)$$

The pinning matrix P associated with the augmented graph $\widetilde{G}$ represents the information flow between the leader and followers, defined as

$$P = diag\{p_1, p_2, \dots, p_N\}. \quad (7)$$

where $p_i = 1$ if edge $\{\alpha_0, \alpha_i\} \in \widetilde{E}$; $p_i = 0$ otherwise. The expression $\{\alpha_0, \alpha_i\} \in \widetilde{E}$ means that vehicle i can receive information from the leader. The weight p_i has been called pinning gains in the field of complex networks [29]. If $p_i = 1$, vehicle i is said to be pinned to the leading vehicle.

Several definitions associated with graph topology $\widetilde{G}$ should be stated for completeness [28]:

1) Directed path. A directed path of length $\zeta + 1$ from node α_i to node α_j is an ordered set of distinct nodes $\{\alpha_i, \alpha_{i_1}, \dots, \alpha_{i_\zeta}, \alpha_j\}$ such that $\{\alpha_i, \alpha_{i_1}\} \in \widetilde{E}, \{\alpha_{i_\zeta}, \alpha_j\} \in \widetilde{E}$ and $\{\alpha_{i_k}, \alpha_{i_{k+1}}\} \in \widetilde{E}$ for all $k \in \{1,2, \dots, \zeta - 1\}$ and $\zeta < N$.
2) Spanning tree. A spanning tree is a tree formed by some or all the edges of graph that connect all the nodes of the graph. The graph $\widetilde{G}$ is said to have a spanning tree if a subset of the edges forms a spanning tree.
3) Neighbor set. Vehicle j is said to be a neighbor of vehicle i if $m_{ij} = 1$, which means vehicle i can obtain information from vehicle j by V2V communication or by radar-based detection. The neighbor set of vehicle i is denoted by $\mathbb{N}_i = \{j | m_{ij} = 1\}$.

Here, it is assumed that the augmented graph $\widetilde{G}$ contains at least one spanning tree rooting from the leader [30]. In other words, there exists a directed path (not necessarily unique) from the leader to every following vehicle, which implies that every follower can obtain the leader information directly or indirectly. It is obvious that all the information flow topologies shown in Fig. 1 satisfy the assumption of containing a spanning tree.

C. Formation of Closed-Loop Platoon Dynamics

In engineering practice, both vehicle dynamics and platoon controllers can be different from each other, which imply that the platoon is heterogeneous. However, a platoon is often formed with the same-type vehicles, e.g. either trucks or passenger vehicles. In such cases, vehicle dynamics are close to each other, i.e. $A_i = A, B_i = B \ (i = 1,2, \cdots, N)$, and their controllers are designed to be identical. Therefore, it is assumed that the platoon is homogeneous in our study, as in

[7] [9] and [10]. The leading vehicle is considered to be of constant-velocity type, i.e. $s_0 = v_0 t$. The objective of platoon control is to track the speed of the leading vehicle while maintaining a rigid formation governed by the constant distance policy between any two consecutive vehicles, i.e.

$$\begin{cases} v_i(t) = v_0(t) \\ s_{i-1}(t) - s_i(t) = d_{i-1,i} \end{cases}, i = 1,2,\cdots N. \qquad (8)$$

where $d_{i-1,i}$ is the desired constant spacing between vehicle i-1 and vehicle i. There are two major spacing policies for vehicular platoons: the constant distance (CD) policy and constant time headway (CTH) policy [3][15]. In the CD policy, the desired distance between two consecutive vehicles is independent of vehicle velocity, which can lead to a very high traffic capacity. For the CTH policy, the desired inter-vehicle range varies with vehicle velocity, which accords with driver behaviors to some extent but limits the achievable traffic capacity. Here, we only consider the CD policy, which means that the vehicles are controlled to move in a rigid formation while following a leading vehicle. Note that $d_{i-1,i}$ contains the length of the vehicle body.

The controllers are distributed in each vehicle, and each controller can only use its neighborhood information specified by the neighbor set $\mathbb{N}_i$. The linear control law of each vehicle is:

$$u_i(t) = -\sum_{j\in\mathbb{N}_i}\big[k_1\big(s_i - s_j - d_{i,j}\big) + k_2\big(v_i - v_j\big) + k_3\big(a_i - a_j\big)\big]. \qquad (9)$$

where $k_\#$ ($\# = 1,2,3$) is the control gain of the linear controller. The desired trajectory of the i-th vehicle is

$$s_i^* = s_0 - d_{0,i} = s_0 - \sum_{j=0}^{i-1} d_{j,j+1}. \qquad (10)$$

For convenience, we define three new tracking errors $\tilde{s}_i$, $\tilde{v}_i$ and $\tilde{a}_i$

$$\begin{cases} \tilde{s}_i = s_i - s_i^* \\ \tilde{v}_i = v_i - \dot{s}_i^* = v_i - v_0 \\ \tilde{a}_i = a_i - \ddot{s}_i^* = a_i \end{cases} \qquad (11)$$

For each vehicle, we can lump its tracking error with neighborhood vehicles specified by $\mathbb{N}_i$. The lumped tracking error is

$$\varepsilon_i = \sum_{j\in\mathbb{N}_i}\big(\tilde{x}_i - \tilde{x}_j\big). \qquad (12)$$

where $\tilde{x}_i = [\tilde{s}_i, \tilde{v}_i, \tilde{a}_i]$. Substituting (12) into (9), the control law is rewritten into a compact form:

$$u_i(t) = -k^T \varepsilon_i(t), \qquad (13)$$

where $k = [k_1, k_2, k_3]^T$. Then, the closed-loop dynamics of vehicle i becomes

$$\begin{aligned} \dot{\tilde{x}}_i &= A\tilde{x}_i - Bk^T\varepsilon_i(t) \\ &= A\tilde{x}_i - Bk^T\big[\sum_{j=1}^N m_{ij}\big(\tilde{x}_i - \tilde{x}_j\big) + p_i(\tilde{x}_i - \tilde{x}_0)\big]. \end{aligned} \qquad (14)$$

For the closed-loop dynamics of the homogeneous platoon, we define the collective states of all vehicles as

$$X = [\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_N]^T. \qquad (15)$$

Hence, the unified overall close-loop dynamics of the platoon interconnected by a given information exchange topology are written in the following compact form

$$\dot{X} = \{I_N \otimes A - (L + P) \otimes Bk^T\}X. \qquad (16)$$

where I_N is the identity matrix and symbol $\otimes$ is the Kronecker product. The overall closed-loop system matrix is

$$A_c = I_N \otimes A - (L + P) \otimes Bk^T. \qquad (17)$$

From (16), it is clear that the platoon dynamics are a function of vehicle longitudinal dynamics (noted by A, B), the information flow topologies (noted by matrix $L + P$), decentralized feedback control law (noted by controller gain k^T) and the spacing policy (noted by $d_{i-1,i}$). The overall closed-loop system matrix A_c, shown in (17), reflects the local vehicle closed-loop matrix $A - Bk^T$ as modified on the information flow topology $L + P$. Therefore, the platoon stability depends on not only its decentralized controllers but also the information flow topologies. Moreover, the information flow can cast fundamental limitation for certain platoon properties, i.e. stability and scalability. In Section III, the stability under different information flow topologies will be analyzed based on (16) through the algebraic graph theory and Routh–Hurwitz stability criterion.

III. Closed Loop Stability of Platoon with Different Information Flow topologies

This section focuses on the stability analysis of homogeneous platoons in a rigid formation. It should be noted that there are two kinds of stability for platoons, i.e.

(1) **Closed-loop stability,** which is measured by the real part of the least stable eigenvalue of the closed-loop matrix A_c.
(2) **String stability,** which is to guarantee the spacing error between consecutive vehicles do not amplify along the vehicle string [4][7].

This paper only considers the closed-loop stability under different information topologies and leaves the string stability for future discussion. Before presenting the main result on closed-loop stability, we need the following Lemmas.

Lemma 1. [31] *Let a matrix $Q = \big[q_{ij}\big] \in R^{n\times n}$. Then all the eigenvalues of Q are located in the union of the n disks*
$$\bigcup_{i=1}^n\big\{\lambda \in \mathbb{C}||\lambda - q_{ii}| \leq \sum_{j=1,j\neq i}^n|q_{ij}|\big\}.$$

Lemma 2. [32] *Let a matrix $Q = \big[q_{ij}\big] \in R^{n\times n}$ and*
$$J = \big\{i \in \{1,2,\cdots,n\}||q_{ii}| > \sum_{j=1,j\neq i}^n|q_{ij}|\big\} \neq \emptyset.$$
If for each $i \notin J$, there is a sequence of nonzero elements of Q of the form $\{q_{ii_1}, q_{i_1i_2}, \cdots, q_{i_rj}\}$ with $j \in J$, then Q is nonsingular.

Lemma 3. [24] *Let $\lambda_i, i = 1,2,\ldots,N$ be the eigenvalues of $L + P$, which may or may not be distinct, Platoon (16) is asymptotically stable if and only if all*
$$A - \lambda_i Bk^T, i = 1,2,\cdots,N \qquad (18)$$
are asymptotically stable.

Lemma 1 is the well-known Geršgorin Disk Criterion. The main result of this paper is stated as follows.

Theorem 1. *Consider the homogeneous platoon with linear controllers given by (16):*
(1.1) All the eigenvalues of $L + P$ are located in the open right-half plane, i.e. $Re\big(\lambda_i(L + P)\big) > 0, i = 1,2,\ldots,N$, when graph $\tilde{G}$ contains a spanning tree.
(1.2) All the eigenvalues of $L + P$ are real, i.e. $\lambda_i(L + P) = Re\big(\lambda_i(L + P)\big), i = 1,2,\ldots,N$, if graph $\tilde{G}$ satisfies one

of the following conditions, no matter how many followers are pinned to the leader:

a) Followers in a platoon are of "look-ahead" type, i.e. each following vehicle can obtain the information of its "k" preceding vehicles.

b) Followers in a platoon are of symmetric "look-ahead & look-back" type, i.e. each following vehicle has the information of its both "k" preceding vehicles and "k" succeeding vehicles.

c) Information flow among followers in a platoon is undirected, i.e. $j \in \mathbb{N}_i \Leftrightarrow i \in \mathbb{N}_j$.

(1.3) If graph $\tilde{G}$ satisfies conditions (1.1) and (1.2), platoon (16) is asymptotically stable if and only if

$$\begin{cases} k_1 > 0 \\ k_2 > k_1\tau/\min(\lambda_i k_3 + 1). \\ k_3 > -1/\max(\lambda_i) \end{cases} \quad (19)$$

Proof: From the definition of Laplacian matrix L in (6), we have

$$\begin{cases} \sum_{j=1}^{N} l_{ij} = 0 \\ |l_{ii}| = \sum_{j=1, j \neq i}^{N} |l_{ij}| \geq 0 \end{cases} \quad (20)$$

Considering the definition of pinning matrix P, we have $p_i \geq 0$ and there is at least one vehicle that can obtain information from the leader because $\tilde{G}$ at least contains a spanning tree. This means that there is at least one node r such that $p_r = 1$. Hence, for matrix $L + P$, we have

$$|l_{ii} + p_i| = |l_{ii}| + |p_i| \geq \sum_{j=1, j \neq i}^{N} |l_{ij}|. \quad (21)$$

By Lemma 1, all the eigenvalues of $L + P$ are located in the union of N disks

$$\cup_{i=1}^{N} \{\lambda \in \mathbb{C} | |\lambda - l_{ii} - p_i| \leq \sum_{j=1, j \neq i}^{N} |l_{ij}|\}. \quad (22)$$

Then, the range of all the eigenvalues of $L + P$ lies in the disk

$$\{\lambda \in \mathbb{C} | |\lambda - \max(l_{ii} + p_i)| \leq \max(l_{ii} + p_i)\}. \quad (23)$$

Hence, all the eigenvalues of $L + P$ lie within the union

$$\{\lambda \in \mathbb{C} | \text{Re}(\lambda) > 0 \cup \{0\}\}. \quad (24)$$

In addition, inequality (21) holds strictly at least for vehicle r, which is pinned to the leader, i.e. $p_r = 1$. Considering the spanning tree assumption, for any vehicle i, which does not have a direct connection to the leader, there must be a direct path connecting vehicle r and vehicle i. Therefore, $L + P$ is nonsingular according to Lemma 2, which implies that all the eigenvalues of $L + P$ are located in the open right-half plane by combining (24), i.e. $\text{Re}(\lambda_i(L + P)) > 0, i = 1,2, \ldots, N$.

To prove **Theorem 1.2**, note that L under assumption (a) is a lower triangular matrix, and P is a diagonal matrix, so $L + P$ is always a lower triangular matrix, which implies that all the eigenvalues of $L + P$ are real, i.e.

$$\lambda_i(L + P) = l_{ii} + p_i, \qquad i = 1,2, \cdots, N \quad (25)$$

Meanwhile, L under assumption (b) or assumption (c) is a symmetric matrix, and hence $L + P$ is also symmetric, which implies that all the eigenvalues of $L + P$ are real, i.e. $\lambda_i(L + P) = \text{Re}(\lambda_i(L + P)), i = 1,2, \ldots, N$.

To prove **Theorem 1.3**, we know that all the eigenvalues of $L + P$ are positive real, by combining the results in the first two statements. According to Lemma 3, Platoon (16) is asymptotically stable if and only if the real parts of the

eigenvalues of matrices $A - \lambda_i B k^T, i = 1,2, \cdots, N$ are all negative. The characteristic polynomial of matrix $A - \lambda_i B k^T$ is

$$|sI - (A - \lambda_i B k^T)| = s^3 + \frac{\lambda_i k_3 + 1}{\tau} s^2 + \frac{\lambda_i k_2}{\tau} s + \frac{\lambda_i k_1}{\tau}. \quad (26)$$

The stability of (26) is studied using the Routh–Hurwitz stability criterion, shown in (27).

$$\begin{array}{ccc} s^3 & 1 & \dfrac{\lambda_i k_2}{\tau} \\[2mm] s^2 & \dfrac{\lambda_i k_3 + 1}{\tau} & \dfrac{\lambda_i k_1}{\tau} \\[2mm] s^1 & \dfrac{\lambda_i k_2(\lambda_i k_3 + 1) - \lambda_i k_1 \tau}{\tau(\lambda_i k_3 + 1)} & \\[2mm] s^0 & \dfrac{\lambda_i k_1}{\tau} & \end{array} \quad (27)$$

Given the fact $\tau > 0, \lambda_i > 0, i = 1,2, \ldots, N$, (26) is asymptotically stable if and only if

$$\begin{cases} k_1 > 0 \\ k_2 > k_1\tau/(\lambda_i k_3 + 1). \\ k_3 > -1/\lambda_i \end{cases} \quad (28)$$

Thus, $A - \lambda_i B k^T, i = 1,2, \cdots, N$ are all asymptotically stable, i.e. system (16) is asymptotically stable if and only if (19) are satisfied. ■

Remark 1: For **Theorem 1.1**, similar results were established in [24][33][34]. The proof in [33] relies on the fact that $L + P$ is irreducible when graph $\tilde{G}$ contains a spanning tree. The technique used in this paper is similar to [34].

Remark 2: In a platoon, if vehicle acceleration is inaccessible, i.e. $k_3 = 0$, then Platoon (16) is asymptotically stable if and only if

$$k_1 > 0, k_2 > k_1\tau. \quad (29)$$

Earlier development of platoons is radar-based, which lacks acceleration information of other vehicles. In such cases, as long as k_1, k_2 satisfy (29), the closed-loop stability of the platoon can be guaranteed.

Remark 3: The conclusion (29) is consistent with [17][18]. In [17] and [18], similar results were obtained using partial differential equation approximation, which is only suitable for platoons with bidirectional and bidirectional-leader topologies. The proof here extends their results and is suitable for a large class of information flow topologies as long as they satisfy conditions (a), (b) and (c) in **Theorem 1**. The conditions cover all aforementioned topologies in Fig. 1

IV. SIMULATION RESULTS

Numerical simulations are conducted to validate the main result. We consider a homogeneous platoon with 11 identical vehicles (1 leader and 10 followers) interconnected by the six information flow topologies shown in Fig. 1 The acceleration or deceleration of the leader can be viewed as disturbances in a platoon. The initial state of the leader is set as $s_0(t) = 0, v_0 = 20 \, m/s$, and the desired trajectory is given by

$$v_0 = \begin{cases} 20 \, m/s & t \leq 5 \, s \\ 20 + 2t \, m/s & 5s < t \leq 10 \, s \, . \\ 30 \, m/s & t > 10s \end{cases}$$

The eigenvalues of associated matrix $L + P$ for these six information flow topologies are listed in TABLE I. All the eigenvalues are positive real, which is consistent with **Theorems 1.1** and **1.2**.

TABLE I.   EIGENVALUES FOR $L + P$ OF DIFFERENT INFORMATION FOW TOPOLOGIES IN FIG. 1 ($N = 10$)

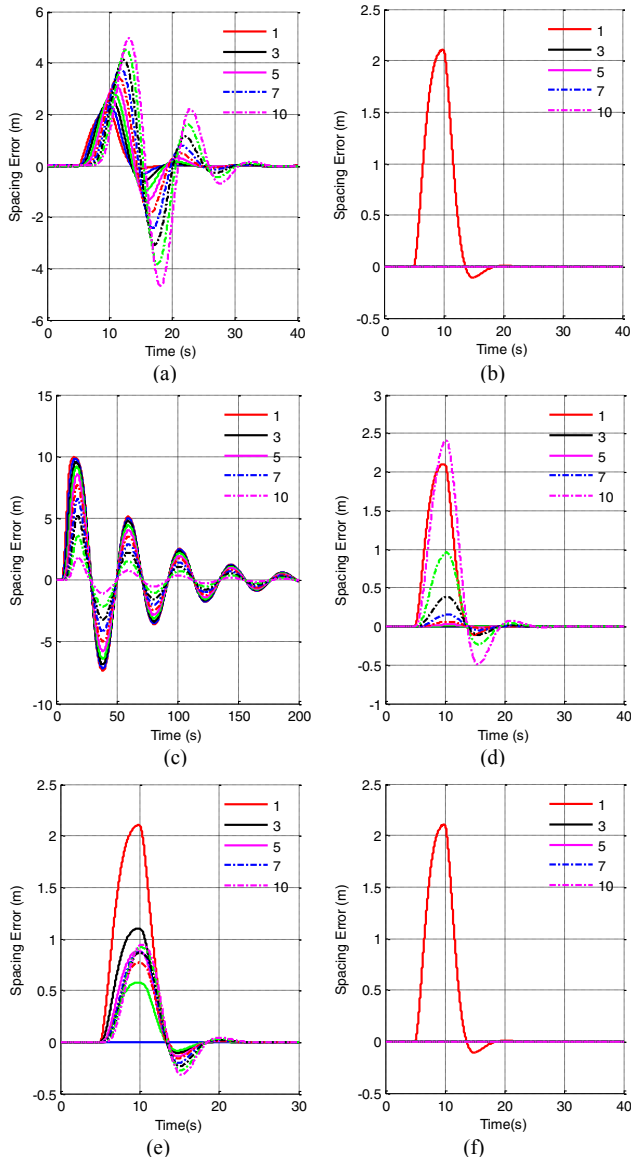| (a) PF	(b) PLF	(c) BD	(d) BDL	(e)TPF	(f)TPLF
1	1	0.0223	1.0000	1	1
1	2	0.1981	1.0979	2	2
1	2	0.5339	1.3820	2	3
1	2	1.0000	1.8244	2	3
1	2	1.5550	2.3820	2	3
1	2	2.1495	3.0000	2	3
1	2	2.7307	3.6180	2	3
1	2	3.2470	4.1756	2	3
1	2	3.6525	4.6180	2	3
1	2	3.9111	4.9021	2	3

Figure 2.   Performance of stable platoon when Theorem 1.3 is satisfied. (a): PF; (b): PLF; (c):BD; (d):BDL; (e): TPF; (f):TPLF
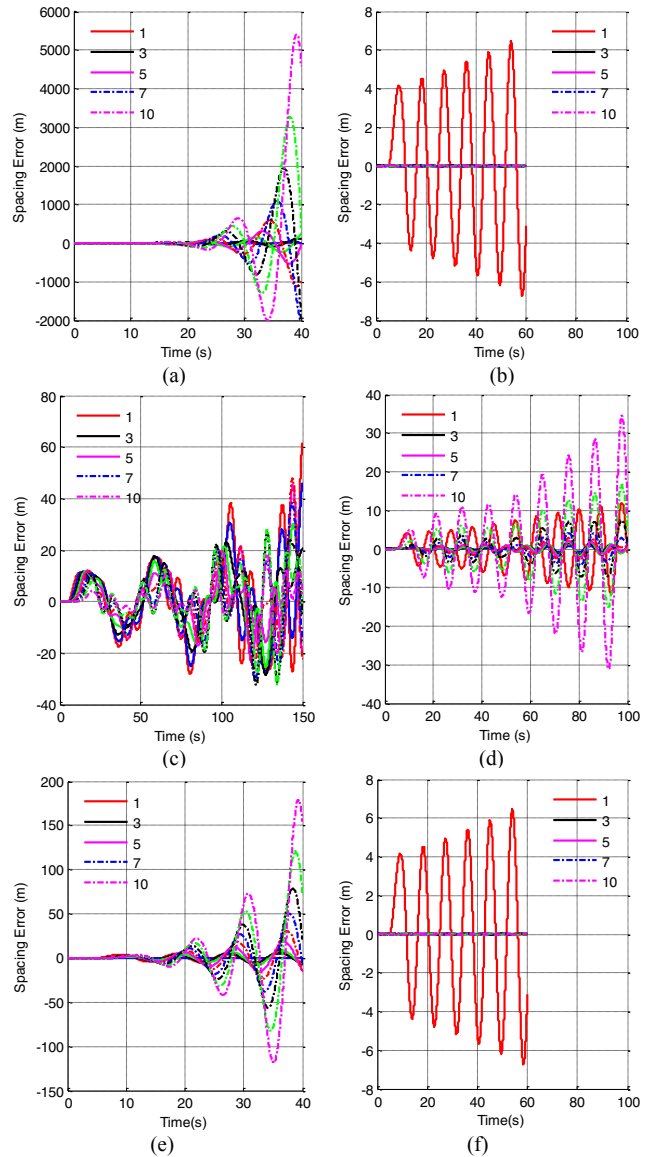

Figure 3.   Performance of unstable platoon when Theorem 1.3 is dissatisfied. (a): PF; (b): PLF; (c):BD; (d):BDL; (e): TPF; (f):TPLF

TABLE II.   PRAMETERS FOR THE PLATOON

Parameters	Scenario 1	Scenario 2
τ	0.4	0.4
k_1	1	1
k_2	2	0.2
k_3	1	1
Theorem 1.3	Satisfied	Dissatisfied

In the simulations, the desired spacing is set as $d_{i-1,i} = 20\ m$ and the vehicle length is equal to $4\ m$. The initial state of the platoon is set as the desired state, i.e. the initial spacing errors and velocity errors are all equal to 0. Two scenarios, i.e. stability and instability, have been simulated by considering two groups of specific parameters (see TABLE II.). Fig. 2 demonstrates spacing errors for different information flow topologies (i.e. Fig. 1 (a)-(f)) in Scenario 1, whose parameters are listed in TABLE II. As the parameters in Scenario 1

satisfy the closed-loop stability condition (19), i.e. **Theorem1.3** holds, the motion of the vehicles is stable for all the information topologies. On the other hand, the parameters in Scenario 2 do not satisfy the stability condition (19), so instability occurs. Considering this fact, Fig. 3 shows the instability of the platoon.

V. CONCLUSIONS

This paper studies the influence of information flow topology on the closed-loop stability of homogeneous vehicular platoons moving in a rigid formation. Using the exact feedback linearization, a linearized vehicle longitudinal dynamic model is derived which takes into account the inertial delay of powertrain dynamics. Directed graph topologies are employed to model allowable information flow among vehicles, both radar-based and communication-based. Linear distributed controllers are designed, leading to platoon closed-loop dynamics under the constant distance policy. The main result explicitly derives the closed-loop stability conditions for platoons with a large class of information flow topologies. Some open questions are worth investigation in this field: (1) the unified closed-loop stability theorem of heterogeneous platoons with non-identical controllers; (2) the unified string stability theorem of platoons under different types of information flow topologies.

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